លំហាត់លីមីតសម្រាប់ថ្នាក់ទី១២

រៀបរៀងដោយ៖ ហឿង សិរីវឌ្ឍន៍

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លំហាត់.១

គណនាលីមីត $\lim\limits_{x\to -1} \dfrac{x^2-1}{x+1}$

$$\begin{aligned}
&\lim_{x\to -1} \frac{x^2-1}{x+1}\\
&= \lim_{x\to -1} \frac{(x-1)(x+1)}{x+1}\\
&= \lim_{x\to -1} (x-1)\\
&= -1-1\\
&= -2
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to -1} \dfrac{x^2-1}{x+1} = -2$

ចំណាំ

- ដើម្បីគណនាលីមីតរាង $\frac{0}{0}$ គេត្រូវដាក់ភាគយកនិងភាគបែងជាផលគុណកត្តា រួចសម្រួលកត្តារួមចោលដើម្បីឲ្យបាត់រាងមិនកំណត់ $\frac{0}{0}$ ។
- $a^2-b^2=(a-b)(a+b)$

លំហាត់.២

គណនាលីមីត $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4x+4}$

$$\lim_{x\to 2} \frac{x-2}{x^2-4x+4}$$
$$= \lim_{x\to 2} \frac{x-2}{(x-2)(x-2)}$$
$$= \lim_{x\to 2} \frac{1}{x-2}$$
$$= \frac{1}{2-2}$$
$$= \frac{1}{0}$$
$$= \infty$$

ដូចនេះ $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4x+4} = \infty$

ចំណាំ

- $a^2-2ab+b^2=(a-b)^2$
- មួយចំនួនចែកដោយសូន្យគឺស្មើរអនន្ត។
- មួយចំនួនចែកដោយអនន្តគឺស្មើរសូន្យ។
  (eg. $\dfrac{1}{10000} = 0.0001 \approx 0$)

លំហាត់.៣

គណនាលីមីត $\lim\limits_{x\to 5} \dfrac{x-5}{x^2-25}$

$$\begin{aligned}
&\lim_{x\to 5} \frac{x-5}{x^2-25} \\
&= \lim_{x\to 5} \frac{x-5}{(x-5)(x+5)} \\
&= \lim_{x\to 5} \frac{1}{x+5} \\
&= \frac{1}{5+5} \\
&= \frac{1}{10}
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 5} \dfrac{x-5}{x^5-25} = \dfrac{1}{10}$

លំហាត់.៤

គណនាលីមីត $\lim\limits_{x\to -2} \dfrac{x^3+8}{x+2}$

ចំណាំ

- $a^3+b^3=(a+b)(a^2-ab+b^2)$
- $a^3-b^3=(a-b)(a^2+ab+b^2)$

$$\begin{aligned}
&\lim_{x\to -2} \frac{x^3+8}{x+2}\\
&= \lim_{x\to -2} \frac{(x+2)(x^2-2x+4)}{x+2}\\
&= \lim_{x\to -2} (x^2-2x+4)\\
&= 4+4+4\\
&= 12
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to -2} \dfrac{x^3+8}{x+2} = 12$

លំហាត់.៥

គណនាលីមីត $\lim\limits_{x\to -1}\dfrac{2x^2-x-3}{x+1}$

$$\lim_{x\to -1}\frac{2x^2-x-3}{x+1}$$
$$=\lim_{x\to -1}\frac{2x^2+2x-3x-3}{x+1}$$
$$=\lim_{x\to -1}\frac{2x(x+1)-3(x+1)}{x+1}$$
$$=\lim_{x\to -1}\frac{(x+1)(2x-3)}{x+1}$$
$$=\lim_{x\to -1}(2x-3)$$
$$=2(-1)-3$$
$$=-5$$

ដូចនេះ $\lim\limits_{x\to -1}\dfrac{2x^2-x-3}{x+1}=-5$

ចំណាំ

វិធី បំបែក តួ កណ្តាល(បំបែក តួb)ដើម្បី ដាក់ជាផលគុណកត្តានៃ $2x^2-x-3$ មានរាង ax^2+bx+c

- យក $2\times(-3)=a\times c=-6$
  $-6=-6\times 1=-3\times 2=\ldots$
- យក $-6=-3\times 2$ ព្រោះ
  $-3+2=-1=b$

យើងបាន

$$2x^2-x-3$$
$$=2x^2+2x-3x-3$$
$$=(2x^2+2x)-(3x+3)$$
$$=2x(x+1)-3(x+1)$$
$$=(x+1)(2x-3)$$

លំហាត់.៦

គណនាលីមីត $\lim\limits_{x\to 2}\dfrac{2-x}{x^2-4}$

$$\begin{aligned}
&\lim_{x\to 2}\frac{2-x}{x^2-4}\\
&=\lim_{x\to 2}\frac{2-x}{(x-2)(x+2)}\\
&=\lim_{x\to 2}\frac{-(x-2)}{(x-2)(x+2)}\\
&=\lim_{x\to 2}\frac{-1}{x+2}\\
&=\frac{-1}{4}
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 2}\dfrac{2-x}{x^2-4}=-\dfrac{1}{4}$

លំហាត់.៧

គណនាលីមីត $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x^2-1}$

$$\begin{aligned}
&\lim_{x\to 1}\frac{x^2+x-2}{x^2-1}\\
&=\lim_{x\to 1}\frac{x^2-x+2x-2}{(x-1)(x+1)}\\
&=\lim_{x\to 1}\frac{x(x-1)+2(x-1)}{(x-1)(x+1)}\\
&=\lim_{x\to 1}\frac{(x-1)(x+2)}{(x-1)(x+1)}\\
&=\lim_{x\to 1}\frac{x+2}{x+1}\\
&=\frac{3}{2}
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x^2-1}=\dfrac{3}{2}$

លំហាត់.៨

គណនាលីមីត $\lim\limits_{x\to+\infty}\dfrac{x+1}{2x+3}$

$$\lim_{x\to+\infty}\frac{x+1}{2x+3}$$
$$=\lim_{x\to+\infty}\frac{x(1+\frac{1}{x})}{x(2+\frac{3}{x})}$$
$$=\lim_{x\to+\infty}\frac{1+\frac{1}{x}}{2+\frac{3}{x}}$$
$$=\frac{1+0}{2+0}$$
$$=\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to+\infty}\dfrac{x+1}{2x+3}=\dfrac{1}{2}$

ចំណាំ

ដើម្បី គណនា លីមីត រាង មិនកំណត់ $\frac{\infty}{\infty}$ យើង ត្រូវ៖

- ចាប់តួណាដែលមានដឺក្រេធំជាងគេឬ សន្ទុះលឿនជាងគេជាផលគុណកត្តា ទាំងភាគយកនិងភាគបែង។
- សម្រួលកត្តារួមចោលដើម្បីបាត់រាង មិនកំណត់។
- គណនាលីមីតនៃកន្សោមថ្មី។

លំហាត់.៩

គណនាលីមីត $\lim\limits_{x\to+\infty} \dfrac{(x+2)(2x+3)}{x^2+1}$

$$\lim_{x\to+\infty} \frac{(x+2)(2x+3)}{x^2+1}$$
$$= \lim_{x\to+\infty} \frac{x^2(1+\frac{2}{x})(2+\frac{3}{x})}{x^2(1+\frac{1}{x^2})}$$
$$= \lim_{x\to+\infty} \frac{(1+\frac{2}{x})(2+\frac{3}{x})}{(1+\frac{1}{x^2})}$$
$$= \frac{(1+0)(2+0)}{1+0}$$
$$= 2$$

ដូចនេះ $\lim\limits_{x\to+\infty} \dfrac{(x+2)(2x+3)}{x^2+1} = 2$

លំហាត់.១០

គណនាលីមីត $\lim\limits_{x\to 1}\dfrac{2x^2+3x-5}{(x-1)(x+2)}$

$$\lim_{x\to 1}\frac{2x^2+3x-5}{(x-1)(x+2)}$$
$$=\lim_{x\to 1}\frac{2x^2-2x+5x-5}{(x-1)(x+2)}$$
$$=\lim_{x\to 1}\frac{2x(x-1)+5(x-1)}{(x-1)(x+2)}$$
$$=\lim_{x\to 1}\frac{(x-1)(2x+5)}{(x-1)(x+2)}$$
$$=\lim_{x\to 1}\frac{2x+5}{x+2}$$
$$=\frac{7}{3}$$

ដូចនេះ $\lim\limits_{x\to 1}\dfrac{2x^2+3x-5}{(x-1)(x+2)}=\dfrac{7}{3}$

លំហាត់.១១

គណនាលីមីត $\lim\limits_{x\to -2} \dfrac{x^3+8}{x^2-4}$

$$\lim_{x\to -2} \frac{x^3+8}{x^2-4}$$
$$= \lim_{x\to -2} \frac{(x+2)(x^2-2x+4)}{(x-2)(x+2)}$$
$$= \lim_{x\to -2} \frac{x^2-2x+4}{x-2}$$
$$= \frac{4+4+4}{-4}$$
$$= -3$$

ដូចនេះ $\lim\limits_{x\to -2} \dfrac{x^3+8}{x^2-4} = -3$

លំហាត់.១២

គណនាលីមីត $\lim\limits_{x\to -1} \dfrac{3x^2-2x-5}{(x+1)(x-4)}$

$$\lim_{x\to -1} \frac{3x^2-2x-5}{(x+1)(x-4)}$$
$$= \lim_{x\to -1} \frac{3x^2+3x-5x-5}{(x+1)(x-4)}$$
$$= \lim_{x\to -1} \frac{3x(x+1)-5(x+1)}{(x+1)(x-4)}$$
$$= \lim_{x\to -1} \frac{(x+1)(3x-5)}{(x+1)(x-4)}$$
$$= \lim_{x\to -1} \frac{3x-5}{x-4}$$
$$= \frac{8}{5}$$

ដូចនេះ $\lim\limits_{x\to -1} \dfrac{3x^2-2x-5}{(x+1)(x-4)} = \dfrac{8}{5}$

លំហាត់.១៣

គណនាលីមីត $\lim\limits_{x\to 1}\dfrac{3x^4-x^2-2}{x^3+2x^2-3}$

$$\begin{aligned}
&\lim_{x\to 1}\frac{3x^4-x^2-2}{x^3+2x^2-3}\\
&=\lim_{x\to 1}\frac{(x-1)(3x^3+3x^2+2x+2)}{(x-1)(x^2+3x+3)}\\
&=\lim_{x\to 1}\frac{3x^3+3x^2+2x+2}{x^2+3x+3}\\
&=\frac{3+3+4}{1+3+3}\\
&=\frac{10}{7}
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 1}\dfrac{3x^4-x^2-2}{x^3+2x^2-3}=\dfrac{10}{7}$

លំហាត់.១៤

គណនាលីមីត $\lim\limits_{x\to 2}\dfrac{x^3-x^2+x-6}{(x+1)(x^2-4)}$

$$\lim_{x\to 2}\frac{x^3-x^2+x-6}{(x+1)(x^2-4)}$$
$$=\lim_{x\to 2}\frac{(x-2)(x^2+x+3)}{(x+1)(x-2)(x+2)}$$
$$=\lim_{x\to 2}\frac{x^2+x+3}{(x+1)(x+2)}$$
$$=\frac{9}{12}$$
$$=\frac{3}{4}$$

ដូចនេះ $\lim\limits_{x\to 2}\dfrac{x^3-x^2+x-6}{(x+1)(x^2-4)}=\dfrac{3}{4}$

លំហាត់.១៥

គណនាលីមីត $\lim\limits_{x\to\infty} \dfrac{x^5+(x+5)^5}{x^5+5}$

$$\begin{aligned}
&\lim_{x\to\infty} \frac{x^5+(x+5)^5}{x^5+5}\\
&= \lim_{x\to\infty} \frac{x^5+x^5(1+\frac{5}{x})^5}{x^5+5}\\
&= \lim_{x\to\infty} \frac{x^5[1+(1+\frac{5}{x})^5]}{x^5(1+\frac{5}{x^5})}\\
&= \lim_{x\to\infty} \frac{[1+(1+\frac{5}{x})^5]}{(1+\frac{5}{x^5})}\\
&= \frac{1+1}{1}\\
&= 2
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to\infty} \dfrac{x^5+(x+5)^5}{x^5+5} = 2$

លំហាត់.១៦

គណនាលីមីត $\lim\limits_{x\to+\infty} \dfrac{\sqrt{4x^2+5x+1}}{x}$

$$\lim_{x\to+\infty} \frac{\sqrt{4x^2+5x+1}}{x}$$
$$= \lim_{x\to+\infty} \frac{x\sqrt{4+\frac{5}{x}+\frac{1}{x^2}}}{x}$$
$$= \lim_{x\to+\infty} \sqrt{4+\frac{5}{x}+\frac{1}{x^2}}$$
$$= \sqrt{4}$$
$$= 2$$

ដូចនេះ $\lim\limits_{x\to+\infty} \dfrac{\sqrt{4x^2+5x+1}}{x} = 2$

លំហាត់.១៧

គណនាលីមីត $\lim\limits_{x\to 1}\dfrac{\sqrt{x+1}-\sqrt{2}}{x^2-1}$

$$\lim_{x\to 1}\frac{\sqrt{x+1}-\sqrt{2}}{x^2-1}$$
$$=\lim_{x\to 1}\frac{(\sqrt{x+1}-\sqrt{2})(\sqrt{x+1}+\sqrt{2})}{(x^2-1)(\sqrt{x+1}+\sqrt{2})}$$
$$=\lim_{x\to 1}\frac{\sqrt{(x+1)^2}-\sqrt{2^2}}{(x^2-1)(\sqrt{x+1}+\sqrt{2})}$$
$$=\lim_{x\to 1}\frac{x+1-2}{(x-1)(x+1)(\sqrt{x+1}+\sqrt{2})}$$
$$=\lim_{x\to 1}\frac{x-1}{(x-1)(x+1)(\sqrt{x+1}+\sqrt{2})}$$
$$=\lim_{x\to 1}\frac{1}{(x+1)(\sqrt{x+1}+\sqrt{2})}=\frac{1}{4\sqrt{2}}$$

ដូចនេះ $\lim\limits_{x\to 1}\dfrac{\sqrt{x+1}-\sqrt{2}}{x^2-1}=\dfrac{1}{4\sqrt{2}}$

លំហាត់.១៨

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}-1}{x}$

$$\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$$
$$=\lim_{x\to 0}\frac{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}{x(\sqrt{x+1}+1)}$$
$$=\lim_{x\to 0}\frac{\sqrt{(x+1)^2}-1}{x(\sqrt{x+1}+1)}$$
$$=\lim_{x\to 0}\frac{x+1-1}{x(\sqrt{x+1}+1)}$$
$$=\lim_{x\to 0}\frac{x}{x(\sqrt{x+1}+1)}$$
$$=\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}-1}{x}=\dfrac{1}{2}$

លំហាត់.១៩

គណនាលីមីត $\lim\limits_{x\to 1}\dfrac{1-\sqrt[3]{2-x}}{1-x}$

$$\lim_{x\to 1}\frac{1-\sqrt[3]{2-x}}{1-x}$$
$$=\lim_{x\to 1}\frac{(1-\sqrt[3]{2-x})(1+\sqrt[3]{2-x}+\sqrt[3]{(2-x)^2})}{(1-x)(1+\sqrt[3]{2-x}+\sqrt[3]{(2-x)^2})}$$
$$=\lim_{x\to 1}\frac{1-2+x}{(1-x)(1+\sqrt[3]{2-x}+\sqrt[3]{(2-x)^2})}$$
$$=\lim_{x\to 1}\frac{-(1-x)}{(1-x)(1+\sqrt[3]{2-x}+\sqrt[3]{(2-x)^2})}$$
$$=\lim_{x\to 1}\frac{-1}{(1+\sqrt[3]{2-x}+\sqrt[3]{(2-x)^2})}$$
$$=-\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to 1}\dfrac{1-\sqrt[3]{2-x}}{1-x}=-\dfrac{1}{3}$

លំហាត់.២0

គណនាលីមីត $\lim\limits_{x\to+\infty}(\sqrt[3]{x^3+6x^2}-x)$

$$\lim_{x\to+\infty}(\sqrt[3]{x^3+6x^2}-x)$$
$$=\lim_{x\to+\infty}\frac{(\sqrt[3]{x^3+6x^2}-x)(\sqrt[3]{(x^3+6x^2)^2}+x\sqrt[3]{x^3+6x^2}+x^2)}{\sqrt[3]{(x^3+6x^2)^2}+x\sqrt[3]{x^3+6x^2}+x^2}$$
$$=\lim_{x\to+\infty}\frac{\sqrt[3]{(x^3+6x^2)^3}-x^3}{\sqrt[3]{(x^3+6x^2)^2}+x\sqrt[3]{x^3+6x^2}+x^2}$$
$$=\lim_{x\to+\infty}\frac{x^3+6x^2-x^3}{x^2\left(\sqrt[3]{(1+\frac{6}{x})^2}+\sqrt[3]{1+\frac{6}{x}}+1\right)}$$
$$=\lim_{x\to+\infty}\frac{6}{\left(\sqrt[3]{(1+\frac{6}{x})^2}+\sqrt[3]{1+\frac{6}{x}}+1\right)}=\frac{6}{3}=2$$

លំហាត់.២១

គណនាលីមីត $\lim\limits_{x\to-\infty}\dfrac{\sqrt{9x^2+3x+1}}{x}$

ចំណាំ

- $\sqrt{x^2}=x$ បើ $x>0$
- $\sqrt{x^2}=-x$ បើ $x<0$

$$\lim_{x\to-\infty}\frac{\sqrt{9x^2+3x+1}}{x}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{x^2\left(9+\dfrac{3}{x}+\dfrac{1}{x^2}\right)}}{x}$$

$$=\lim_{x\to-\infty}\frac{-x\sqrt{9+\dfrac{3}{x}+\dfrac{1}{x^2}}}{x}$$

$$=-3$$

ដូចនេះ $\lim\limits_{x\to-\infty}\dfrac{\sqrt{9x^2+3x+1}}{x}=-3$

លំហាត់.២២

គណនាលីមីត $\lim\limits_{x\to+\infty}(\sqrt{x+\sqrt{x}}-\sqrt{x})$

$$\lim_{x\to+\infty}(\sqrt{x+\sqrt{x}}-\sqrt{x})$$

$$=\lim_{x\to+\infty}\frac{(\sqrt{x+\sqrt{x}}-\sqrt{x})(\sqrt{x+\sqrt{x}}+\sqrt{x})}{\sqrt{x+\sqrt{x}}+\sqrt{x}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{(x+\sqrt{x})^2}-\sqrt{x^2}}{\sqrt{x+\sqrt{x}}+\sqrt{x}}=\lim_{x\to+\infty}\frac{x+\sqrt{x}-x}{\sqrt{x+\sqrt{x}}+\sqrt{x}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}+\sqrt{x}}=\lim_{x\to+\infty}\frac{1}{\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x}}+1}$$

$$=\lim_{x\to+\infty}\frac{1}{\sqrt{\frac{x+\sqrt{x}}{x}}+1}==\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{\sqrt{x}}{x}}+1}=\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to+\infty}(\sqrt{x+\sqrt{x}}-\sqrt{x})=\frac{1}{2}$

លំហាត់.២៣

គណនាលីមីត $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{(x-1)(2x+3)}$

$$\lim_{x\to 1}\frac{\sqrt{x+3}-2}{(x-1)(2x+3)}$$
$$=\lim_{x\to 1}\frac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(2x+3)(\sqrt{x+3}+2)}$$
$$=\lim_{x\to 1}\frac{\sqrt{(x+3)^2}-2^2}{(x-1)(2x+3)(\sqrt{x+3}+2)}$$
$$=\lim_{x\to 1}\frac{x+3-4}{(x-1)(2x+3)(\sqrt{x+3}+2)}$$
$$=\lim_{x\to 1}\frac{x-1}{(x-1)(2x+3)(\sqrt{x+3}+2)}$$
$$=\lim_{x\to 1}\frac{1}{(2x+3)(\sqrt{x+3}+2)}$$
$$=\frac{1}{20}$$

លំហាត់.២៤

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x+1}-1}{x(x+2)}$

$$\lim_{x\to 0}\frac{\sqrt[3]{x+1}-1}{x(x+2)}$$
$$=\lim_{x\to 0}\frac{(\sqrt[3]{x+1}-1)(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1)}{x(x+2)(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1)}$$
$$=\lim_{x\to 0}\frac{\sqrt[3]{(x+1)^3}-1}{x(x+2)(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1)}$$
$$=\lim_{x\to 0}\frac{x+1-1}{x(x+2)(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1)}$$
$$=\lim_{x\to 0}\frac{1}{(x+2)(\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1)}$$
$$=\frac{1}{6}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x+1}-1}{x(x+2)}=\dfrac{1}{6}$

លំហាត់.២៥

គណនាលីមីត $\lim\limits_{x\to 2}\dfrac{x-\sqrt{x+2}}{\sqrt{4x+1}-3}$

$$\lim_{x\to 2}\frac{x-\sqrt{x+2}}{\sqrt{4x+1}-3}$$
$$=\lim_{x\to 2}\frac{(\sqrt{4x+1}+3)(x-\sqrt{x+2})(x+\sqrt{x+2})}{(\sqrt{4x+1}+3)(\sqrt{4x+1}-3)(x+\sqrt{x+2})}$$
$$=\lim_{x\to 2}\frac{(\sqrt{4x+1}+3)(x^2-\sqrt{(x+2)^2})}{(\sqrt{(4x+1)^2}-3^2)(x+\sqrt{x+2})}$$
$$=\lim_{x\to 2}\frac{(\sqrt{4x+1}+3)(x^2-x-2)}{(4x+1-9)(x+\sqrt{x+2})}$$
$$=\lim_{x\to 2}\frac{(x-2)(x+1)(\sqrt{4x+1}+3)}{4(x-2)(x+\sqrt{x+2})}$$
$$=\lim_{x\to 2}\frac{(x+1)(\sqrt{4x+1}+3)}{4(x+\sqrt{x+2})}$$
$$=\frac{3\times 6}{4\times 4}=\frac{9}{8}$$

លំហាត់.២៦

គណនាលីមីត $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+1}-x\right)$

$$\lim_{x\to+\infty}\left(\sqrt{x^2+1}-x\right)$$
$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)}{\sqrt{x^2+1}+x}$$
$$=\lim_{x\to+\infty}\frac{\sqrt{(x^2+1)^2}-x^2}{\sqrt{x^2+1}+x}$$
$$=\lim_{x\to+\infty}\frac{x^2+1-x^2}{\sqrt{x^2+1}+x}$$
$$=\lim_{x\to+\infty}\frac{1}{\sqrt{x^2+1}+x}$$
$$=0$$

ដូចនេះ $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+1}-x\right)=0$

លំហាត់.២៧

គណនាលីមីត $\lim\limits_{x\to+\infty}\dfrac{x-\sqrt{x^2-1}}{\sqrt{x^2-4}}$

$$\lim_{x\to+\infty}\frac{x-\sqrt{x^2-1}}{\sqrt{x^2-4}}$$

$$=\lim_{x\to+\infty}\frac{x-x\sqrt{1-\dfrac{1}{x^2}}}{x\sqrt{1-\dfrac{4}{x^2}}}$$

$$=\lim_{x\to+\infty}\frac{x\left(1-\sqrt{1-\dfrac{1}{x^2}}\right)}{x\sqrt{1-\dfrac{4}{x^2}}}=\frac{1-1}{1}=0$$

ដូចនេះ $\lim\limits_{x\to+\infty}\dfrac{x-\sqrt{x^2-1}}{\sqrt{x^2-4}}=0$

លំហាត់.២៨

គណនាលីមីត $\lim\limits_{x\to 2}\dfrac{x-2}{\sqrt{x^2-2}-\sqrt{2}}$

$$\lim_{x\to 2}\frac{x-2}{\sqrt{x^2-2}-\sqrt{2}}$$
$$=\lim_{x\to 2}\frac{(x-2)(x+2)(\sqrt{x^2-2}+\sqrt{2})}{(x+2)(\sqrt{x^2-2}-\sqrt{2})(\sqrt{x^2-2}+\sqrt{2})}$$
$$=\lim_{x\to 2}\frac{(x^2-2^2)(\sqrt{x^2-2}+\sqrt{2})}{(x+2)(\sqrt{(x^2-2)^2}-\sqrt{2^2})}$$
$$=\lim_{x\to 2}\frac{(x^2-4)(\sqrt{x^2-2}+\sqrt{2})}{(x+2)(x^2-2-2)}$$
$$=\lim_{x\to 2}\frac{\sqrt{x^2-2}+\sqrt{2}}{x+2}$$
$$=\frac{2\sqrt{2}}{4}=\frac{\sqrt{2}}{2}$$

ដូចនេះ $\lim\limits_{x\to 2}\dfrac{x-2}{\sqrt{x^2-2}-\sqrt{2}}=\dfrac{\sqrt{2}}{2}$

លំហាត់.២៩

គណនាលីមីត $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x-1}-\sqrt{x^2+1}\right)$

$$\lim_{x\to+\infty}\left(\sqrt{x^2+x-1}-\sqrt{x^2+1}\right)$$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2+x-1}-\sqrt{x^2+1}\right)\left(\sqrt{x^2+x-1}+\sqrt{x^2+1}\right)}{\sqrt{x^2+x-1}+\sqrt{x^2+1}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{(x^2+x-1)^2}-\sqrt{(x^2+1)^2}}{\sqrt{x^2(1+\frac{1}{x}-\frac{1}{x^2})}+\sqrt{x^2(1+\frac{1}{x^2})}}$$

$$=\lim_{x\to+\infty}\frac{x^2+x-1-x^2-1}{x\left(\sqrt{1+\frac{1}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)}$$

$$=\lim_{x\to+\infty}\frac{1-\frac{2}{x}}{\sqrt{1+\frac{1}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}}=\frac{1}{2}$$

លំហាត់.៣០

គណនាលីមីត $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+2x}-\sqrt[3]{x^3-3x^2}\right)$

$$\lim_{x\to+\infty}\left(\sqrt{x^2+2x}-\sqrt[3]{x^3-3x^2}\right)$$
$$=\lim_{x\to+\infty}\left(\sqrt{x^2+2x}-x+x-\sqrt[3]{x^3-3x^2}\right)$$
$$=\lim_{x\to+\infty}\left(\sqrt{x^2+2x}-x\right)-\lim_{x\to+\infty}\left(\sqrt[3]{x^3-3x^2}-x\right)$$
$$=\lim_{x\to+\infty}\frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x}-\lim_{x\to+\infty}\frac{x^3-3x^2-x^3}{\sqrt[3]{(x^3-3x^2)^2}+x\sqrt[3]{x^3-3x^2}+x^2}$$
$$=\lim_{x\to+\infty}\frac{2x}{x\sqrt{1+\frac{2}{x}}+x}-\lim_{x\to+\infty}\frac{-3x^2}{x^2\sqrt[3]{(1-\frac{3}{x})^2}+x^2\sqrt[3]{1-\frac{3}{x}}+x^2}$$
$$=\lim_{x\to+\infty}\frac{2}{\sqrt{1+\frac{2}{x}}+1}-\lim_{x\to+\infty}\frac{-3}{\sqrt[3]{(1-\frac{3}{x})^2}+\sqrt[3]{1-\frac{3}{x}}+1}$$
$$=\frac{2}{2}+\frac{3}{3}=1+1=2$$

លំហាត់.៣១

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{2\sin x}{3x}$

$$\begin{aligned}&\lim_{x\to 0}\frac{2\sin x}{3x}\\ &=\frac{2}{3}\lim_{x\to 0}\frac{\sin x}{x}\\ &=\frac{2}{3}\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{2\sin x}{3x}=\dfrac{2}{3}$

ចំណាំ

- $\lim\limits_{x\to 0}\dfrac{\sin ax}{ax}=\lim\limits_{x\to 0}\dfrac{ax}{\sin ax}=1$
- $\lim\limits_{x\to 0}\dfrac{\tan ax}{ax}=\lim\limits_{x\to 0}\dfrac{ax}{\tan ax}=1$

លំហាត់.៣២

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{1-\cos x}{2x}$

$$\lim_{x\to 0}\frac{1-\cos x}{2x}$$
$$=\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{2x(1+\cos x)}$$
$$=\lim_{x\to 0}\frac{1-\cos^2 x}{2x(1+\cos x)}$$
$$=\lim_{x\to 0}\frac{\sin^2 x}{2x(1+\cos x)}$$
$$=\lim_{x\to 0}\frac{\sin^2 x}{x^2}\times\frac{x}{2(1+\cos x)}$$
$$=1\times\frac{0}{2(1+1)}=0$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{1-\cos x}{2x}=0$

ចំណាំ

- $(a-b)(a+b)=a^2-b^2$
- $\sin^2 x+\cos^2 x=1$

លំហាត់.៣៣

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{x+\sin x}{2x-3\sin 2x}$

$$\lim_{x\to 0}\frac{x+\sin x}{2x-3\sin 2x}$$
$$=\lim_{x\to 0}\frac{x\left(1+\dfrac{\sin x}{x}\right)}{x\left(2-\dfrac{6\sin 2x}{2x}\right)}$$
$$=\lim_{x\to 0}\frac{1+\dfrac{\sin x}{x}}{2-\dfrac{6\sin 2x}{2x}}$$
$$=\frac{1+1}{2-6}=\frac{2}{-4}=-\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{x+\sin x}{2x-3\sin 2x}=-\dfrac{1}{2}$

លំហាត់.៣៤

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{\sin x+2x}{x-2\sin x}$

$$\lim_{x\to 0}\frac{\sin x+2x}{x-2\sin x}$$
$$=\lim_{x\to 0}\frac{x\left(\dfrac{\sin x}{x}+2\right)}{x\left(1-\dfrac{2\sin x}{x}\right)}$$
$$=\lim_{x\to 0}\frac{\dfrac{\sin x}{x}+2}{1-\dfrac{2\sin x}{x}}$$
$$=\frac{1+2}{1-2}=-3$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\sin x+2x}{x-2\sin x}=-3$

លំហាត់.៣៥

គណនាលីមីត $\lim\limits_{x\to 0} \dfrac{1-\cos^2 x}{3x^2}$

$$\begin{aligned}
&\lim_{x\to 0} \frac{1-\cos^2 x}{3x^2} \\
&= \lim_{x\to 0} \frac{\sin^2 x}{3x^2} \\
&= \frac{1}{3} \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 \\
&= \frac{1}{3}
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 0} \dfrac{1-\cos^2 x}{3x^2} = \dfrac{1}{3}$

លំហាត់.៣៦

គណនាលីមីត $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\pi-2x}{\cos x}$

តាង $t=\pi-2x$ នោះ $x=\dfrac{\pi}{2}-\dfrac{t}{2}$

បើ $x\to\dfrac{\pi}{2}$ នោះ $t\to 0$

យើងបាន

$\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\pi-2x}{\cos x}$

$$= \lim_{t\to 0} \frac{t}{\cos(\frac{\pi}{2}-\frac{t}{2})} = \lim_{t\to 0} \frac{t}{\sin\frac{t}{2}} = 2\lim_{t\to 0} \frac{\frac{t}{2}}{\sin\frac{t}{2}} = 2$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\pi-2x}{\cos x} = 2$

ចំណាំ

- $\cos(\dfrac{\pi}{2}-\alpha)=\sin\alpha$
- $\sin(\dfrac{\pi}{2}-\alpha)=\cos\alpha$

លំហាត់.៣៧

គណនាលីមីត $\lim\limits_{x\to 1}\dfrac{\sin \pi x}{x-1}$

តាង $t=x-1$ នោះ $x=t+1$
បើ $x\to 1$ នោះ $t\to 0$
យើងបាន

$$\lim_{x\to 1}\frac{\sin \pi x}{x-1}$$
$$=\lim_{t\to 0}\frac{\sin \pi(t+1)}{t}=\lim_{t\to 0}\frac{\sin(\pi+\pi t)}{t}=\lim_{t\to 0}\frac{-\sin \pi t}{t}=-\pi$$

ដូចនេះ $\lim\limits_{x\to 1}\dfrac{\sin \pi x}{x-1}=-\pi$

ចំណាំ

- $\sin(\pi+\alpha)=-\sin\alpha$
- $\cos(\pi+\alpha)=-\cos\alpha$
- $\sin(\pi-\alpha)=\sin\alpha$
- $\cos(\pi-\alpha)=-\cos\alpha$

លំហាត់.៣៨

គណនាលីមីត $\lim\limits_{x\to 1}\dfrac{1-\sin\dfrac{\pi x}{2}}{(1-x)^2}$

តាង $t=1-x$ នោះ $x=1-t$
បើ $x\to 1$ នោះ $t\to 0$
យើងបាន

$$\lim_{x\to 1}\frac{1-\sin\frac{\pi x}{2}}{(1-x)^2}=\lim_{t\to 0}\frac{1-\sin\frac{\pi}{2}(1-t)}{t^2}=\lim_{t\to 0}\frac{1-\sin\left(\frac{\pi}{2}-\frac{\pi}{2}t\right)}{t^2}$$

$$=\lim_{t\to 0}\frac{1-\cos\frac{\pi}{2}t}{t^2}=\lim_{t\to 0}\frac{\sin^2\frac{\pi}{2}t}{t^2\left(1+\cos\frac{\pi}{2}t\right)}$$

$$=\lim_{t\to 0}\left(\frac{\sin\frac{\pi}{2}t}{\frac{\pi}{2}t}\right)^2\times\frac{\left(\frac{\pi}{2}\right)^2}{1+\cos\frac{\pi}{2}t}=\frac{\pi^2}{8}$$

លំហាត់.៣៩

គណនាលីមីត $\lim\limits_{x\to 1}(1-x^2)\tan\dfrac{\pi x}{2}$

តាង $t=1-x$ នោះ $x=1-t$
បើ $x\to 1$ នោះ $t\to 0$
យើងបាន

ចំណាំ

- $\lim\limits_{x\to 0}\dfrac{\sin ax}{ax}=\lim\limits_{x\to 0}\dfrac{ax}{\sin ax}=1$
- $\lim\limits_{x\to 0}\dfrac{\tan ax}{ax}=\lim\limits_{x\to 0}\dfrac{ax}{\tan ax}=1$

$$\lim_{x\to 1}(1-x^2)\tan\frac{\pi x}{2}$$
$$=\lim_{x\to 1}(1-x)(1+x)\tan\frac{\pi}{2}x$$
$$=\lim_{t\to 0}t(2-t)\tan\frac{\pi}{2}(1-t)=\lim_{t\to 0}t(2-t)\tan\left(\frac{\pi}{2}-\frac{\pi}{2}t\right)$$
$$=\lim_{t\to 0}\frac{\frac{\pi}{2}t}{\tan\frac{\pi}{2}t}\times\frac{2-t}{\frac{\pi}{2}}=\frac{4}{\pi}$$

ដូចនេះ $\lim\limits_{x\to 1}(1-x^2)\tan\dfrac{\pi}{2}x=\dfrac{4}{\pi}$

លំហាត់.៤0

គណនាលីមីត $\lim\limits_{x\to\pi}\dfrac{1-\cos 2x}{(\pi-x)^2}$

$$\lim_{x\to\pi}\frac{1-\cos 2x}{(\pi-x)^2}=\lim_{x\to\pi}\frac{2\sin^2 x}{(\pi-x)^2}$$

តាង $t=\pi-x$ នោះ $x=\pi-t$
បើ $x\to\pi$ នោះ $t\to 0$
យើងបាន

$$\lim_{x\to\pi}\frac{2\sin^2 x}{(\pi-x)^2}=\lim_{t\to 0}\frac{2\sin^2(\pi-t)}{t^2}$$

$$=\lim_{t\to 0}\frac{2\sin^2 t}{t^2}=2\lim_{t\to 0}\left(\frac{\sin t}{t}\right)^2=2$$

ដូចនេះ $\lim\limits_{x\to\pi}\dfrac{1-\cos 2x}{(\pi-x)^2}=2$

ចំណាំ

- $\cos 2x=1-2\sin^2 x$
- $\sin(\pi-\alpha)=\sin\alpha$

លំហាត់.៤១

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x\sin x}$

$$\begin{aligned}&\lim_{x\to 0}\frac{1-\cos 2x}{x\sin x}\\ &=\lim_{x\to 0}\frac{2\sin^2 x}{x\sin x}\\ &=\lim_{x\to 0}\frac{2\sin x}{x}\\ &=2\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x\sin x}=2$

ចំណាំ

- $\cos 2x = 1-2\sin^2 x$
- $\lim\limits_{x\to 0}\dfrac{\sin ax}{ax}=1$

លំហាត់.៤២

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{\sin 4x+\sin 7x}{\sin 3x}$

$$\lim_{x\to 0}\frac{\sin 4x+\sin 7x}{\sin 3x}$$
$$=\lim_{x\to 0}\frac{x\left(\dfrac{4\sin 4x}{4x}+\dfrac{7\sin 7x}{7x}\right)}{3x\times\dfrac{\sin 3x}{3x}}$$
$$=\frac{4+7}{3}$$
$$=\frac{11}{3}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\sin 4x+\sin 7x}{\sin 3x}=\dfrac{11}{3}$

លំហាត់.៤៣

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}}{\sin x}$

ចំណាំ

- $\lim\limits_{x\to 0}\dfrac{e^{ax}-1}{ax}=1$
- $\lim\limits_{x\to 0}\dfrac{ax}{e^{ax}-1}=1$

$$\begin{aligned}
&\lim_{x\to 0}\frac{e^x-e^{-x}}{\sin x}\\
&=\lim_{x\to 0}\frac{e^x-1+1-e^{-x}}{\sin x}\\
&=\lim_{x\to 0}\frac{e^x-1}{\sin x}-\lim_{x\to 0}\frac{e^{-x}-1}{\sin x}\\
&=\lim_{x\to 0}\frac{e^x-1}{x}\times\frac{x}{\sin x}-\lim_{x\to 0}\frac{e^{-x}-1}{-x}\times\frac{-x}{\sin x}\\
&=1-(-1)\\
&=2
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}}{\sin x}=2$

លំហាត់.៤៤

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{e^{-x^2}-\cos 2x}{x^2}$

ចំណាំ

- $\lim\limits_{x\to 0}\dfrac{e^{f(x)}-1}{f(x)}=1$
  កាលណា$x\to 0$នោះ$f(0)\to 0$

$$\lim_{x\to 0}\frac{e^{-x^2}-\cos 2x}{x^2}$$
$$=\lim_{x\to 0}\frac{e^{-x^2}-1+1-\cos 2x}{x^2}$$
$$=\lim_{x\to 0}\frac{e^{-x^2}-1}{x^2}+\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$$
$$=-1+\lim_{x\to 0}\frac{\sin^2 2x}{4x^2}\times\frac{4}{1+\cos 2x}$$
$$=-1+2\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^2$$
$$=-1+2=1$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{e^{-x^2}-\cos 2x}{x^2}=1$

លំហាត់.៤៥

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{e^{ax}-e^{bx}}{x}$

$$\begin{aligned}
&\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x}\\
&=\lim_{x\to 0}\frac{e^{ax}-1+1-e^{bx}}{x}\\
&=\lim_{x\to 0}\frac{e^{ax}-1}{x}-\lim_{x\to 0}\frac{e^{bx}-1}{x}\\
&=\lim_{x\to 0}a\times\frac{e^{ax}-1}{ax}-\lim_{x\to 0}b\times\frac{e^{bx}-1}{bx}\\
&=a-b
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{e^{ax}-e^{bx}}{x}=a-b$

លំហាត់.៤៦

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{e^x+e^{2x}-2}{\sin x}$

$$\lim_{x\to 0}\frac{e^x+e^{2x}-2}{\sin x}$$
$$=\lim_{x\to 0}\frac{e^x-1+e^{2x}-1}{\sin x}$$
$$=\lim_{x\to 0}\frac{e^x-1}{x}\times\frac{x}{\sin x}+\lim_{x\to 0}\frac{e^{2x}-1}{2x}\times\frac{2x}{\sin x}$$
$$=1+2=3$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{e^x+e^{2x}-2}{\sin x}=3$

លំហាត់.៤៧

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{e^x+x-1}{x}$

$$\begin{aligned}
&\lim_{x\to 0}\frac{e^x+x-1}{x}\\
&=\lim_{x\to 0}\frac{e^x-1+x}{x}\\
&=\lim_{x\to 0}\left(\frac{e^x-1}{x}+\frac{x}{x}\right)\\
&=\lim_{x\to 0}\frac{e^x-1}{x}+1\\
&=1+1\\
&=2
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{e^x+x-1}{x}=2$

លំហាត់.៤៨

គណនាលីមីត $\lim\limits_{x\to+\infty}(e^x - x^2 + 1)$

$$\lim_{x\to+\infty}(e^x - x^2 + 1)$$
$$= \lim_{x\to+\infty} e^x\left(1 - \frac{x^2}{e^x} + \frac{1}{e^x}\right)$$
$$= +\infty(1 - 0 + 0)$$
$$= +\infty$$

ដូចនេះ $\lim\limits_{x\to+\infty}(e^x - x^2 + 1) = +\infty$

ចំណាំ

- ដើម្បីគណនាលីមីតរាង $\infty - \infty$ គេគ្រាន់តែគណនាលីមីតលើតួណាដែលមានដឺក្រេធំជាងគេឬតួណាដែលមានសន្ទុះលឿនជាងគេ។
- ឬដើម្បីគណនាលីមីតរាង $\infty - \infty$ គេត្រូវចាប់តួណាដែលមានដឺក្រេធំជាងគេឬតួណាដែលមានសន្ទុះលឿនជាងគេជាផលគុណកត្តារួចគណនាលីមីតនោះម្តងទៀត។

លំហាត់.៤៩

គណនាលីមីត $\lim\limits_{x\to-\infty}(e^x+2x^2-1)$

$$\begin{aligned}
&\lim_{x\to-\infty}(e^x+2x^2-1)\\
&=\lim_{x\to-\infty}e^x\left(1+\frac{2x^2}{e^x}-\frac{1}{e^x}\right)\\
&=-\infty(1+0-0)\\
&=-\infty
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to+\infty}(e^x+2x^2-1)=-\infty$

លំហាត់.៥0

គណនាលីមីត $\lim\limits_{x\to+\infty} \dfrac{e^x+2020}{x^2-2x+10}$

$$\lim_{x\to+\infty} \frac{e^x+2020}{x^2-2x+10}$$
$$= \lim_{x\to+\infty} \frac{e^x\left(1+\dfrac{2020}{e^x}\right)}{x^2\left(1-\dfrac{2}{x}+\dfrac{10}{x^2}\right)}$$
$$= \lim_{x\to+\infty} \frac{e^x}{x^2}$$
$$= +\infty$$

ដូចនេះ $\lim\limits_{x\to+\infty} \dfrac{e^x+2020}{x^2-2x+10} = +\infty$

លំហាត់.៥១

គណនាលីមីត $\lim\limits_{x\to+\infty} \dfrac{x^2-3x+2}{e^x+x-20}$

$$\lim_{x\to+\infty} \frac{x^2-3x+2}{e^x+x-20}$$
$$= \lim_{x\to+\infty} \frac{x^2\left(1-\frac{3}{x}+\frac{2}{x^2}\right)}{e^x\left(1+\frac{x}{e^x}-\frac{20}{e^x}\right)}$$
$$= \lim_{x\to+\infty} \frac{x^2}{e^x}$$
$$= 0$$

ដូចនេះ $\lim\limits_{x\to+\infty} \dfrac{x^2-3x+2}{e^x+x-20} = 0$

លំហាត់.៥២

គណនាលីមីត $\lim\limits_{x\to+\infty}\left(1+\dfrac{1}{3x}\right)^x$

$$\lim_{x\to+\infty}\left(1+\frac{1}{3x}\right)^x$$
$$=\lim_{x\to+\infty}\left(1+\frac{1}{3x}\right)^{3x\times\frac{1}{3}}$$
$$=\lim_{x\to+\infty}\left[\left(1+\frac{1}{3x}\right)^{3x}\right]^{\frac{1}{3}}$$
$$=e^{\frac{1}{3}}$$

ដូចនេះ $\lim\limits_{x\to+\infty}\left(1+\dfrac{1}{3x}\right)^x=e^{\frac{1}{3}}$

ចំណាំ

- $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$
- $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{f(x)}\right)^{f(x)}=e$
  កាលណា$x\to\infty$នោះ$f(x)\to\infty$

លំហាត់.៥៣

គណនាលីមីត $\lim\limits_{x\to-\infty} \ln\dfrac{x-1}{x+1}$

ចំណាំ

- $\ln a^x = x\ln a$
- $\ln e = 1$

$$\lim_{x\to-\infty} \ln\frac{x-1}{x+1}$$
$$= \lim_{x\to-\infty} \ln\frac{x+1-2}{x+1}$$
$$= \lim_{x\to-\infty} \ln\left(1+\frac{-2}{x+1}\right)$$
$$= \lim_{x\to-\infty} \ln\left[\left(1+\frac{1}{\frac{x+1}{-2}}\right)^{\frac{x+1}{-2}}\right]^{\frac{-2}{x+1}}$$
$$= \lim_{x\to-\infty} \frac{-2}{x+1}\ln e = 0$$

ដូចនេះ $\lim\limits_{x\to-\infty} \ln\dfrac{x-1}{x+1} = 0$

លំហាត់.៥៤

គណនាលីមីត $\lim\limits_{x\to 0}\frac{1}{x}\ln\sqrt{\frac{1+x}{1-x}}$

ចំណាំ

- $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=e$
- $\lim\limits_{x\to 0}[1+f(x)]^{\frac{1}{f(x)}}=e$
  កាលណា$x\to 0$នោះ$f(x)\to 0$

$$\lim_{x\to 0}\frac{1}{x}\ln\sqrt{\frac{1+x}{1-x}}$$
$$=\lim_{x\to 0}\frac{1}{2x}\ln\frac{1+x}{1-x}$$
$$=\lim_{x\to 0}\frac{1}{2x}\ln\frac{1-x+2x}{1-x}$$
$$=\lim_{x\to 0}\frac{1}{2x}\ln\left[\left(1+\frac{2x}{1-x}\right)^{\frac{1-x}{2x}}\right]^{\frac{2x}{1-x}}$$
$$=\lim_{x\to 0}\frac{1}{2x}\times\frac{2x}{1-x}\ln e=\lim_{x\to 0}\frac{1}{1-x}=1$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{1}{x}\ln\sqrt{\frac{1+x}{1-x}}=1$

លំហាត់.៥៥

គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{\ln(1+3x)}{x}$

$$\lim_{x\to 0}\frac{\ln(1+3x)}{x}$$

$$= \lim_{x\to 0}\frac{\ln(1+3x)^{\frac{1}{3x}\times 3x}}{x}$$

$$= \lim_{x\to 0}\frac{3x\ln(1+3x)^{\frac{1}{3x}}}{x}$$

$$= \lim_{x\to 0}\frac{3x}{x}\ln e$$

$$= 3$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\ln(1+3x)}{x} = 3$

ចំណាំ

- $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}} = e$
- $\lim\limits_{x\to 0}[1+f(x)]^{\frac{1}{f(x)}} = e$
  កាលណា $x\to 0$ នោះ $f(x)\to 0$
- $\ln a^x = x\ln a$
- $\ln e = 1$